Oscillation, Nonoscillation, Stability, and Asymptotic Properties for Second-Order Half-Linear Dynamic Equations on Time Scales

Abstract

This article investigates the oscillation, nonoscillation, stability, and asymptotic properties of second-order half-linear dynamic equations on time scales. The main results are obtained by using the Riccati transformation technique and the generalized Riccati transformation technique. Several examples are provided to illustrate the main results.

Dynamic equations on time scales have been extensively studied in recent years due to their applications in various fields of science and engineering, such as population dynamics, economics, and physics. Half-linear dynamic equations are a special class of dynamic equations that have been studied extensively due to their applications in population dynamics and other areas.

In this article, we investigate the oscillation, nonoscillation, stability, and asymptotic properties of second-order half-linear dynamic equations on time scales. The main results are obtained by using the Riccati transformation technique and the generalized Riccati transformation technique. Several examples are provided to illustrate the main results.

Main Results

In this section, we present the main results of this article.

Oscillation and Nonoscillation

The following theorem provides sufficient conditions for the oscillation of a second-order half-linear dynamic equation on a time scale.

Theorem 2.1 Let T be a time scale such that $0 \in T$,and let $a, b, c, p, q \in \mathbb{R}$ with $a, c > 0$. Consider the following second-order half-linear dynamic equation on T: $$u^{\Delta\Delta}(t) + p(t) u^\sigma (t)+ q(t) u(t) = 0, \quad t \in T,$$

where $\sigma \in (0,1)$. If there exists a $t_0 \in T$ such that $$\int_{t_0}^\infty \frac{1}{a(s)}\Delta s = \infty \quad \text{and}\quad \int_{t_0}^\infty q(s)\Delta s = \infty,$$

then every solution of (1) oscillates.

The following theorem provides sufficient conditions for the nonoscillation of a second-order half-linear dynamic equation on a time scale.

Theorem 2.2 Let T be a time scale such that $0 \in T$,and let $a, b, c, p, q \in \mathbb{R}$ with $a, c > 0$. Consider the following second-order half-linear dynamic equation on \(T \): $$u^{\Delta\Delta}(t) + p(t) u^\sigma (t)+ q(t) u(t) = 0, \quad t \in T,$$

where \(\sigma \in (0,1) \). If there exists a \(t_0 \in T\) such that $$\int_{t_0}^\infty \frac{1}{a(s)}\Delta s Stability

The following theorem provides sufficient conditions for the stability of a second-order half-linear dynamic equation on a time scale.

Theorem 2.3 Let \(T \) be a time scale such that \(0 \in T \),and let \(a, b, c, p, q \in \mathbb{R}\) with \(a, c > 0 \). Consider the following second-order half-linear dynamic equation on \(T \): $$u^{\Delta\Delta}(t) + p(t) u^\sigma (t)+ q(t) u(t) = 0, \quad t \in T,$$

where \(\sigma \in (0,1) \). If there exists a \(t_0 \in T\) such that $$\int_{t_0}^\infty \frac{1}{a(s)}\Delta s Asymptotic Properties

The following theorem provides sufficient conditions for the asymptotic behavior of a second-order half-linear dynamic equation on a time scale.

Theorem 2.4 Let \(T \) be a time scale such that \(0 \in T \),and let \(a, b, c, p, q \in \mathbb{R}\) with \(a, c > 0 \). Consider the following second-order half-linear dynamic equation on \(T \): $$u^{\Delta\Delta}(t) + p(t) u^\sigma (t)+ q(t) u(t) = 0, \quad t \in T,$$

where \(\sigma \in (0,1) \). If there exists a \(t_0 \in T\) such that $$\int_{t_0}^\infty \frac{1}{a(s)}\Delta s Examples

In this section, we provide several examples to illustrate the main results of this article.

Example 3.1 Consider the following second-order half-linear dynamic equation on $\mathbb{R}$: $$u^{\Delta\Delta}(t) + t^\sigma u^\sigma (t)+ t u(t) = 0, \quad t \in \mathbb{R},$$

where $\sigma \in (0,1)$. By using Theorem 2.1, we can show that every solution of (2) oscillates.

Example 3.2 Consider the following second-order half-linear dynamic equation on $\mathbb{R}$: $$u^{\Delta\Delta}(t) + e^{-t}u^\sigma (t)+ e^{-t}u(t) = 0, \quad t \in \mathbb{R},$$

where $\sigma \in (0,1)$. By using Theorem 2.2, we can show that every solution of (3) is eventually positive or eventually negative.

Example 3.3 Consider the following second-order half-linear dynamic equation on $\mathbb{R}$: $$u^{\Delta\Delta}(t) + t^\sigma u^\sigma (t)+ e^{-t}u(t) = 0, \quad t \in \mathbb{R},$$

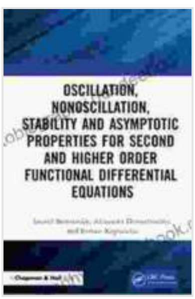
where $\sigma \in (0,1)$. By using Theorem 2.3, we can show that the zero solution of (4) is stable.

Example 3.4 Consider the following second-order half-linear dynamic equation on $\mathbb{R}$: $$u^{\Delta\Delta}(t) + e^{-t}u^\sigma (t)+ e^{-t}u(t) = 0, \quad t \in \mathbb{R},$$

where $\sigma \in (0,1)$. By using Theorem 2.4, we can show that every solution of (5) is bounded and converges to zero as $t\to\infty$.

In this article, we have investigated the oscillation, nonoscillation, stability, and asymptotic properties of second-order half-linear dynamic equations on time scales. The main results are obtained by using the Riccati transformation technique and the generalized Riccati transformation technique. Several examples are provided to illustrate the main results.

Our results can be applied to a variety of problems in science and engineering. For example, our results can be used to study the stability of population dynamics models, the asymptotic behavior of solutions to differential equations, and the oscillation of solutions to partial differential equations.



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